Nonlinear Interpolation on Manifold of Reduced Order Models in Magnetodynamic Problems

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Proper Orthogonal Decomposition (POD) is an efficient model order reduction technique for linear problems in computational sciences, recently gaining popularity in electromagnetics. However, its efficiency has been shown to considerably degrade for nonlinear problems. In this paper, we propose a reduced order model for nonlinear magnetodynamic problems by combining POD with an interpolation on manifolds, which interpolates the reduced bases to efficiently construct the desired solution.

***Index Terms*—Nonlinear Magnetodynamic Problem, Model Order Reduction, Proper Orthogonal Decomposition, Interpolation on Manifolds.**

I. INTRODUCTION

In the model order reduction (MOR) community, the Proper Orthogonal Decomposition (POD) is a popular technique to reduce the number of unknowns of a given problem [1], leading to a new (reduced) model that can be solved at reduced computational cost. When the output variables of interest depend linearly on the input parameters, a single reduced model can be constructed. In the general case however (e.g. for nonlinear magnetodynamic problems), one has to recompute a new reduced model from scratch for each input parameter values.

In this paper, we propose to first apply the POD to construct reduced order models of nonlinear magnetodynamic systems for a discrete set of values of the input parameters. This construction is performed offline, as a (possibly computationally expensive) pre-processing step. Then, for arbitrary values of the input parameters, a new reduced order model is efficiently constructed online by using interpolation on manifolds theory [2]. This reduced model can then be solved in quasi real-time to produce the desired solution. Contrary to other nonlinear MOR techniques [3], [4], the goal here is not to speed up single shot calculations. Rather, we want to pre-construct and store a small library (of reduced models) that can be used later on to obtain very fast solutions of nonlinear problems. As a simple application example, we apply this procedure to a nonlinear inductor-core system, solved using a classical two-dimensional finite element method (FEM).

II. NONLINEAR MAGNETODYNAMIC PROBLEM

A. Formulation

Let us consider a domain Ω of boundary Γ where the problem is solved on $\Omega \times [0,T]$ with T the final computation time. In this problem, the source is imposed directly as a current density $\mathbf{j_s}(t)$ in an inductor. The nonlinearity occurs in the magnetic core, where a typical nonlinear saturation behavior is used. The magnetodynamic problem is formulated in terms of the magnetic vector potential $\mathbf{a}(t)$ such that $\mathbf{b}(t) = \mathbf{curl}\,\mathbf{a}(t)$:

$$\mathbf{curl}\,(\nu\ \mathbf{curl}\,\mathbf{a}(t)) + \sigma\partial_t \mathbf{a}(t) = \mathbf{j_s}(t). \tag{1}$$

In the previous equation $\mathbf{b}$, σ and ν denote the magnetic flux density, the conductivity and the reluctivity, respectively. To tackle the nonlinearity, the fixed point method is used until the residual is sufficiently small [5]. In our case, one can linearize the problem by setting $\nu(\mathbf{curl}\,\mathbf{a}) = \nu_0 + \tilde{\nu}(\mathbf{curl}\,\mathbf{a})$. Equation (1) becomes:

$$\mathbf{curl}\,(\nu_0\ \mathbf{curl}\,\mathbf{a}(t)) + \sigma\partial_t \mathbf{a}(t) - \mathbf{j_s}(t) = -\mathbf{curl}\,(\tilde{\nu}\ \mathbf{curl}\,\mathbf{a}(t)), \tag{2}$$

where the left hand side (LHS) is linear and the right hand side (RHS) is nonlinear. Applying the standard Galerkin FEM leads to the following system of differential algebraic equations [3]:

$$M_1\dot{\mathbf{x}} + M_2\mathbf{x} - B(t) = -M_3(\mathbf{x})\mathbf{x}, \tag{3}$$

where $\mathbf{x} = \mathbf{x}(t)$ is the vector of unknowns, M_1 represents the dynamic behavior, M_2 comes from the linearization around ν_0, B depends on the source current density $\mathbf{j}_s(t)$ and M_3 contains the nonlinear contributions.

B. Euler Scheme

An implicit Euler scheme is chosen for the time discretization, leading to the discrete system of equations:

$$\left(M_1 + \Delta t\left[M_2 + M_3(\mathbf{x}_t^k)\right]\right)\mathbf{x}_t^{k+1} = M_1\mathbf{x}_{t-1} + \Delta t B(t), \tag{4}$$

with $\mathbf{x}_t^{k+1}$ the $k+1$ nonlinear iteration solution at time t. The nonlinear contribution is determined by using $\mathbf{x}_t^k$, the previous nonlinear iteration at the current time step.

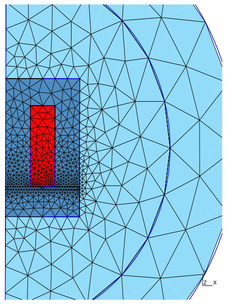

Fig. 1. Inductor-core system FEM model (windings in red, core in dark blue and infinite air region in light blue).

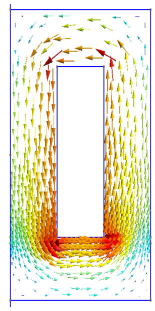

Fig. 2. Distribution of magnetic field.

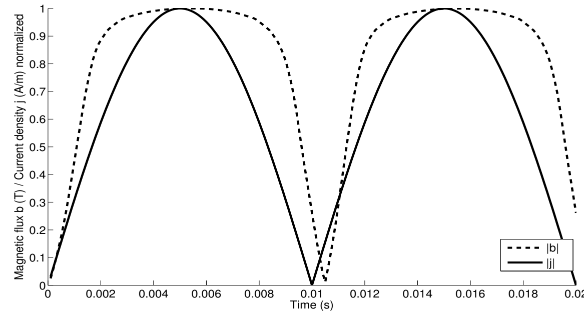

Fig. 3. Normalized values of magnetic field (from nonlinear MOR) and input current vs time.

C. Proper Orthogonal Decomposition

To reduce the system (4), the POD is applied. The vector $\mathbf{x}(t)$ is approximated in a reduced basis by a vector $\mathbf{x_r}(t)$ which satisfies:

$$\mathbf{x} = \Psi \mathbf{x_r}. \tag{5}$$

The projection matrix Ψ is obtained by applying an SVD to snapshots consisting in the solution at each time step. The reduced system then reads:

$$\begin{aligned}\left(M_{r,1} + \Delta t \left[M_{r,2} + M_{r,3}(\Psi \mathbf{x}^k_{r,t})\right]\right) \mathbf{x}^{k+1}_{r,t} \\ = M_{r,1}\mathbf{x}_{r,t-1} + \Delta t B_r(t),\end{aligned} \tag{6}$$

where $M_{r,i} = \Psi^T M_i \Psi$ and $B_r(t) = \Psi^T B(t)$. The nonlinear part still depends on the full order solution; the DEIM can be used to speed up its evaluation [6], [7].

D. Manifold Interpolation of Reduced Order Model

Considering the case where we have calculated the full and reduced solutions for different values of the current source density $\mathbf{j_s}$, we now want to reuse these solutions to efficiently compute the solution for another current source. However, the projection matrix Ψ^* corresponding to this new input parameter is unknown. Because these reduction matrices result from an SVD, they lie on the manifold of orthogonal matrices and can be interpolated. They are mapped to the tangent space at point Q of this manifold using the logarithm mapping [2]

$$\gamma = \mathrm{Log}_Q(P) = \textsc{logm}(Q^T P), \tag{7}$$

and their projections are interpolated to get the projection of Ψ^* (e.g. through a Lagrange interpolation). The new projection is mapped back using the exponential mapping

$$P = \mathrm{Exp}_Q(\gamma) = Q\,\textsc{expm}(\gamma). \tag{8}$$

Details about the interpolation procedure will be provided in the full paper.

III. Results

The approach described above has been applied to a simple nonlinear inductor-core system (Fig. 1) to compute the solution for a previously not considered input current of 10 A. The resulting distribution of the flux density $\mathbf{b}$ in the core is shown in Fig. 2. The full order model has been solved for currents of 5 and 12 A. The relative error between the reduced and the full models for an input current of 10 A is around 1e-4. Directly using the projection matrix corresponding to a previously computed solution (e.g. 5 A) leads to a relative error of 1e-1, and demonstrating the importance of the proposed approach. The nonlinear behavior of the solution obtained after nonlinear MOR is highlighted on Fig. 3.

As a second case test, the full order model has been solved for frequencies 340Hz and 360Hz in the nonlinear state. The spatial distribution changes according to the frequency due to dynamic effects. The relative L2 error between the reduced and the full models for a frequency of 350Hz is around 1% with the orthogonal manifold interpolation. When the same test is conducted with a standard Lagrange interpolation the resulting L2 error between both models is around 200%, which proves the interest of the proposed approach. These results will be investigated further in the full paper.

Acknowledgements

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